OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2603

Pure Mathematics 3

INSTRUCTIONS

Monday **10 JANUARY 2005** Afternoon 1 hour 20 minutes
+ up to 1 hour

The paper is in two parts:

Section A (1 hour 20 minutes)
Section B (up to 1 hour)

Supervisors are requested to ensure that Section B **is not issued** until Section A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Invigilators are not required to match up candidates' two parts. Part A and Part B should be sent to the examiner as two sets of scripts with candidates in the same order as the attendance register for each set.

This notice must be on the Invigilator's desk at all times during the afternoon of Monday 10 January 2005.

These instructions consist of 1 printed page and 1 blank page.

SP (NF) S83690

OCR RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2603(A)

Pure Mathematics 3

Section A

Monday **10 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

NOTE

- This paper will be followed by **Section B: Comprehension.**

This question paper consists of 4 printed pages.

1 **(a)** Express $\dfrac{5}{(x-2)(x^2+1)}$ in the form $\dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+1}$, where A, B and C are constants to be determined. [5]

(b) Write down the exact values of sin 30° and cos 30°. Express $\sin(x+30°)$ in terms of $\sin x$ and $\cos x$.

Hence solve the equation

$$\sqrt{3}\sin x+\cos x=1 \qquad \text{for } -180°<x<180°.$$ [6]

(c) A curve has parametric equations

$$x=1+t^2,\ y=t^3.$$

Find $\dfrac{dy}{dx}$ in terms of t. Hence find the gradient of the curve at the point $(2, 1)$. [4]

2 **(i)** Using a small angle approximation for $\cos x$, show that, for small values of x,

$$x\sec^2 x\approx x(1-\tfrac{1}{2}x^2)^{-2}.$$

Use a binomial expansion to show that, for small x,

$$x\sec^2 x\approx x+x^3.$$ [4]

(ii) Use this result to evaluate $\displaystyle\int_0^{0.1} x\sec^2 x\,dx$ approximately. [2]

(iii) By differentiating $\dfrac{\sin x}{\cos x}$, show that the derivative of $\tan x$ is $\sec^2 x$. [3]

(iv) Use integration by parts, together with the result $\displaystyle\int\tan x\,dx=-\ln\cos x+c$, to evaluate

$$\int_0^{0.1} x\sec^2 x\,dx.$$

Comment on the accuracy of the approximate result you found in part **(ii)**. [6]

3 **(i)** Show that the solution to the differential equation

$$\frac{dy}{dx} = -\frac{4x}{y},$$

with $y = 2$ when $x = 0$, is

$$4x^2 + y^2 = 4.$$ [4]

(ii) Show that parametric equations for the curve $4x^2 + y^2 = 4$ are

$$x = \cos\theta,\ y = 2\sin\theta.$$

What sort of curve is this? [3]

Fig. 3 shows part of the curve, and a point P on it. The tangent at P meets the x-axis at Q. PQ makes an angle β with the x-axis, and angle POQ is α, where $0° < \alpha < 90°$.

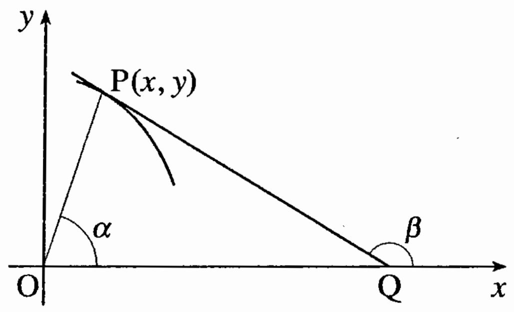

Fig. 3

(iii) Express $\tan\alpha$ in terms of x and y. Hence, using the differential equation given in part **(i)** and the fact that $\frac{dy}{dx} = \tan\beta$, show that

$$\tan\alpha\ \tan\beta = -4.$$ [2]

(iv) You are given that, for a particular point on the curve, $\beta = 2\alpha$. Show that, for this point,

$$\tan^2\alpha = 2.$$

Find the value of α, giving your answer to the nearest degree. [6]

4 Fig. 4 shows the roof of a house. The coordinates of points A, B, C, D, E and F with respect to axes Ox, Oy and Oz are as shown in the diagram. ABCDE is a plane. All lengths are in metres.

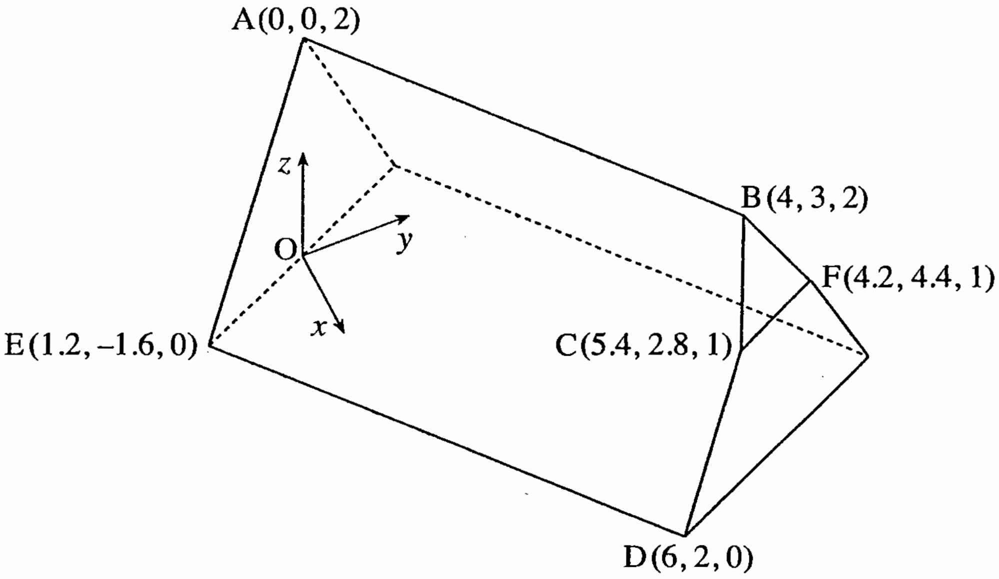

Fig. 4

(i) Write down the vectors $\overrightarrow{BC}$ and $\overrightarrow{BF}$. [1]

(ii) Show that triangle BCF is isosceles. [2]

(iii) Verify that the vector $\mathbf{n}_1 = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ is normal to the plane BCF. Deduce the equation of the plane BCF. [5]

(iv) Verify that the cartesian equation of the plane through A, B and D is

$$3x - 4y + 5z = 10.$$

Write down a vector $\mathbf{n}_2$ normal to this plane. [3]

(v) Find the angle between the planes BCF and ABCDE. [4]

Candidate Name	Centre Number	Candidate Number

RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
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MEI STRUCTURED MATHEMATICS **2603(B)**

Pure Mathematics 3

Section B: Comprehension

Monday **10 JANUARY 2005** Afternoon Up to 1 hour

Additional materials:
Rough paper
MEI Examination Formulae and Tables (MF12)

TIME Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces at the top of this page.
- Answer **all** questions.
- Write your answers in the spaces on the question paper.
- You are permitted to use a graphical calculator in this paper.

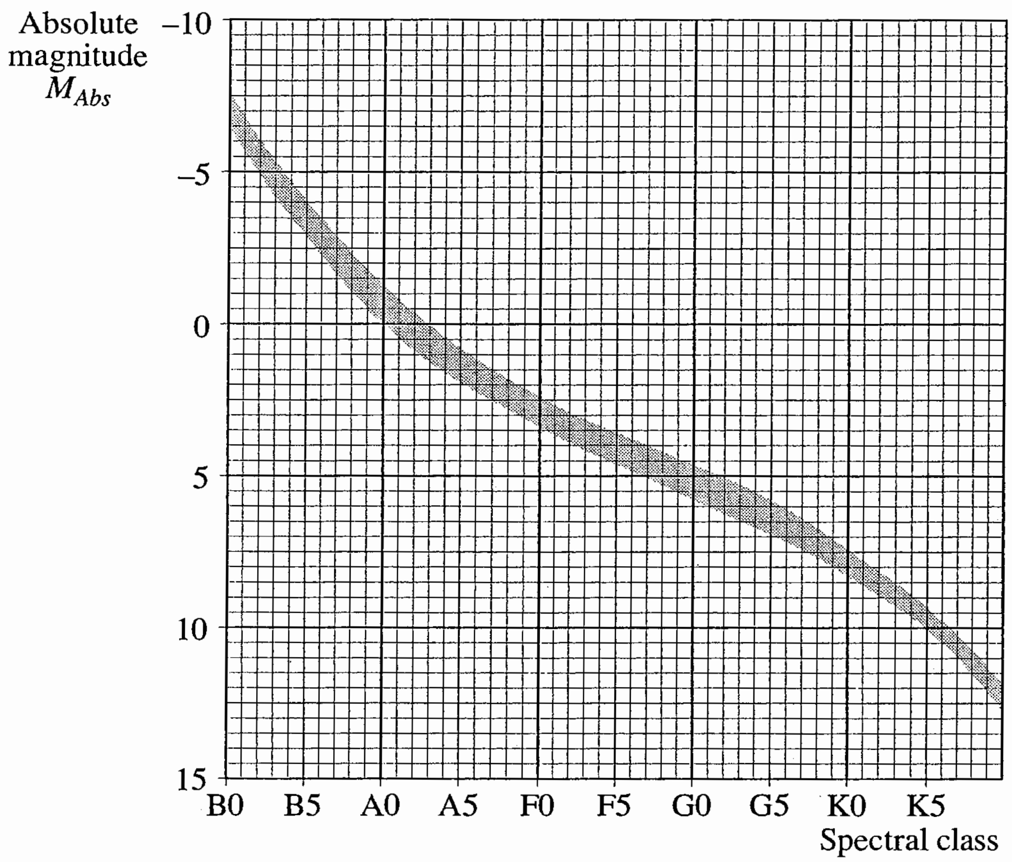
INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this section is 15.

This question paper consists of 4 printed pages and an insert.

For Examiner's Use

1 Using information contained in the article, plot the positions of the Sun, Sirius and Polaris on this HR diagram. [3]



2 The star Bellatrix has apparent magnitude 1.64. The star Arcturus appears to be 4.70 times as bright as Bellatrix.

Use the formula on line 75 to find the apparent magnitude of Arcturus. [2]

....

....

....

....

For Examiner's Use

3 In this question, take the Earth's orbit to be circular with radius 1.5×10^8 km and the speed of light to be 3×10^5 km s^{-1}.

(i) Calculate

(*A*) the number of kilometres in 1 parsec, [3]

....

....

....

(*B*) the number of light years in 1 parsec. [2]

....

....

....

[1 light year is the distance light travels in one year.]

(ii) In line 47, the article says "To 3 significant figures, 1 parsec is 3.26 light years".

Explain why the calculation in part **(i)**(*B*) does not give 3.26. [1]

....

....

4 Show how the result

$$M_{Abs} = M_{App} - 2.5\log_{10}\left(\frac{d^2}{100}\right)$$

on line 100 can be used to derive the result on line 102

$$M_{Abs} = M_{App} + 5 - 5\log_{10} d.$$ [3]

....

....

....

....

For Examiner's Use

5 The brightest star in our night sky, Sirius, is a main sequence star of type A1. Explain how this is possible, given that other stars are higher on the main sequence and thus brighter, for example those of types O and B. [1]

....

....

....

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2603(B)

Pure Mathematics 3

Section B: Comprehension

INSERT

Monday **10 JANUARY 2005** Afternoon Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- This insert contains the text for use with the questions.

This insert consists of 7 printed pages and 1 blank page.

Classifying stars

Introduction

On a really dark, clear night you can see over 2000 stars. Although each star is apparently a pinprick of light, even to the naked eye they do not all look the same.

- Some are brighter than others.
- They vary in colour, with some noticeably redder than others.

These two characteristics, brightness and colour, provide a great deal of information and a basis for classifying stars.

Brightness

There are two reasons for a star to appear bright.

- It is emitting a lot of light.
- It is not far away.

In order to deduce anything from the apparent brightness of a star, it is essential to distinguish between these two causes. This can be done if the distance of the star from us is known. How can such a distance be measured?

Finding distance: the parallax method

Hold up a finger at arm's length and look at it first through one eye and then through the other. Your finger appears to move across the background. Your two eyes see it at somewhat different angles, as illustrated in Fig. 1.

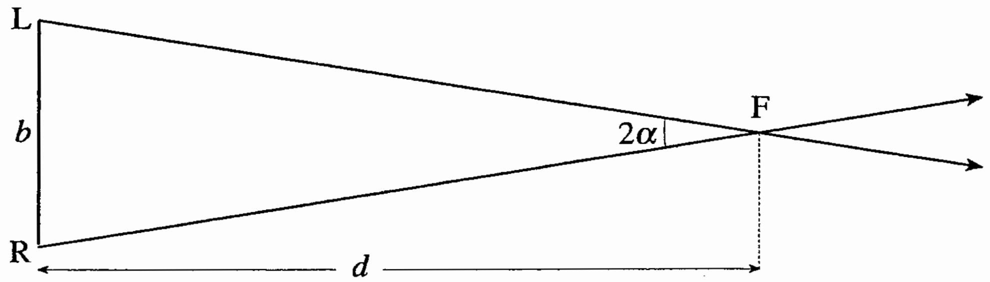


Fig. 1

The points L and R represent your two eyes, and F your finger. The distance between your eyes is b and the distance to your finger is d. The angle between the lines of vision of your eyes is 2α.

The angle α is given by

$$\alpha = \arctan\left(\frac{b}{2d}\right).$$

It represents the apparent angular shift of the object (in this case your finger) from the central position. For a typical adult, the values of b and d are about 6 and 60, in centimetres, giving a value of α of about 3°.

This equation can also be written as

$$d = \frac{b}{2\tan\alpha}.$$

In this form it allows you to calculate d given α and b. This method of finding a distance is called the *parallax method* and α is called the *angle of parallax*.

The parallax method can be used to determine the distance of a star. Fig. 2 shows a star, S, that is reasonably close to us. It is observed from points X and Y. The points X and Y are much further apart than a pair of human eyes! They are the positions of an observer on planet Earth at 6-month intervals. During this time the Earth has completed half an orbit round the sun and so X and Y are at opposite ends of a diameter of this orbit. In this article, for the sake of simplicity, the orbit of the Earth is taken to be circular with radius 150 million kilometres and so the distance b is 3×10^8 km.

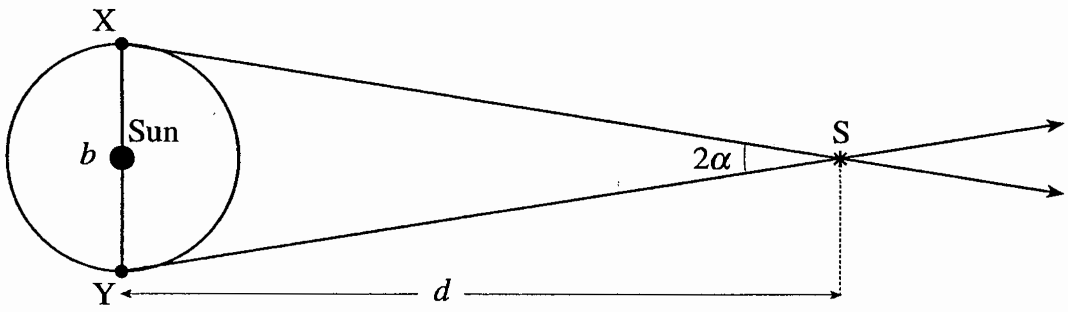
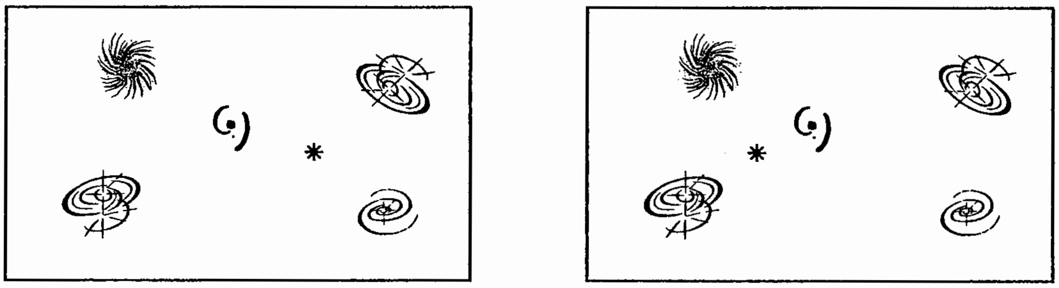


Fig. 2

The two observations of the star show a small movement relative to the background formed by distant stars and galaxies that are so far away that they form a fixed, recognisable pattern. This is illustrated in Fig. 3.



View from X　　　　View from Y

Fig. 3

From this apparent movement of the star, it is possible to determine the angle α and so the distance of the star. In such measurements the angle α will always be small, usually less than one second of arc. (One degree is 60 minutes; one minute is 60 seconds. The symbols for minutes and seconds are ' and " and so $1° = 60', 1' = 60''$.)

In the case when the angle α is one second, the distance, d, of the star is called 1 parsec. The parsec is a widely used unit in astronomy. To 3 significant figures, 1 parsec is 3.26 light years.

The method of parallax works well for nearby stars, but its use is limited by the difficulty in measuring very small angles of parallax from Earth. Even with the best available telescopes, distances beyond about 100 parsecs cannot be measured. That is small compared with the estimated radius of our galaxy, which is 15 000 parsecs. In 1989 the satellite HIPPARCOS was launched specifically to provide more accurate parallax measurements, making it possible to measure distances of up to about 500 parsecs by this method.

Other methods, based on properties of particular types of stars, have been developed for measuring greater distances.

The magnitude of a star

Once you know the distance of a star, it is possible to calculate its brightness, that is the rate at which it is emitting energy. This is known as its *absolute magnitude*. First, however, it is helpful to think of the star's *apparent magnitude*, that is its brightness as seen from the Earth; this is measured by the energy received from it.

Apparent magnitude

The scale used for apparent magnitude is somewhat curious; its origins are historical. The stars visible to the naked eye nearly all fall within the range 0 to 6, but the *smaller* the number the *brighter* the star. The brightest stars have apparent magnitude near 0. (There are in fact a few stars brighter than apparent magnitude 0.) A person with normal eyesight cannot see stars with apparent magnitude greater than about 6. The Pole Star, *Polaris*, has apparent magnitude very close to 2.

The scale is logarithmic with 5 units corresponding to a factor of 100 in brightness.

Thus a star which is 100 times as bright as Polaris would have an apparent magnitude of

$$2 - 5 = -3.$$

(No star in our night sky is actually as bright as this.)

A star which is $\frac{1}{100}$th as bright as Polaris has an apparent magnitude of

$$2 + 5 = 7.$$

This relationship can be expressed by the formula

$$m_2 - m_1 = -2.5 \times \log_{10}\left(\frac{E_2}{E_1}\right).$$

In this formula, m_1 and m_2 are the apparent magnitudes of two stars; E_1 and E_2 are the rates at which we receive energy from them.

In the example above, Polaris can be taken as the star with apparent magnitude m_1 and so $m_1 = 2$. If there were a star 100 times as bright as Polaris, then $\frac{E_2}{E_1} = 100$ and so $m_2 = -3$.

The brightest star in our night sky is actually Sirius with apparent magnitude –1.46.

Absolute magnitude

The absolute magnitude of a star is defined to be the apparent magnitude it would have if it were situated at a distance of 10 parsecs from us.

As an example, think of a star of apparent magnitude 6.0 which is at a distance of 30 parsecs. What is its absolute magnitude?

The amount of energy we receive from a star is inversely proportional to the square of its distance from us. "Moving" a star from 30 parsecs to 10 parsecs changes its distance by a factor of $\frac{1}{3}$. The corresponding factor for the change in the energy received is

$$\frac{1}{\left(\frac{1}{3}\right)^2} = 9.$$

We would receive 9 times the amount of energy and so would see the star as being 9 times as bright in its "new" position.

The next step in the calculation involves using the formula

$$m_2 - m_1 = -2.5 \times \log_{10}\left(\frac{E_2}{E_1}\right).$$

In this case, instead of referring to two different stars, m_1 and m_2 refer to the same star at two different distances from the Earth.

So $m_1 = 6$ and $\frac{E_2}{E_1} = 9$, giving $m_2 = 3.6$. Thus the star in question has absolute magnitude 3.6.

The work in this example can be generalised to give a formula for the absolute magnitude, M_{Abs}, of a star in terms of its apparent magnitude, M_{App}, and its distance, d, in parsecs.

$$M_{Abs} = M_{App} - 2.5\log_{10}\left(\frac{d^2}{100}\right).$$

This can be simplified to give

$$M_{Abs} = M_{App} + 5 - 5\log_{10} d.$$

The absolute magnitudes of Sirius and Polaris are 1.4 and −4.6 respectively; that of the Sun is 4.8.

The colour of a star

A star's colour indicates its temperature because different lines in a star's spectrum are prominent at different temperatures. Red stars, for example, are relatively cool.

Stars' spectra are used to classify them in a sequence, going from hot to cool: O, B, A, F, G, K, M, R, N. The temperature of an O star is typically about 50 000° Kelvin, that of an N star about 3000° Kelvin. (To convert temperatures from Kelvin to Celsius, subtract 273°.)

This sequence was developed towards the end of the 19th century. A group of students at Princeton University soon came up with the mnemonic "Oh Be A Fine Girl Kiss Me Right Now"; this has been used by astronomers around the world ever since. The sequence has now been extended by the discovery of W stars that are even hotter than O stars. At the cool end there are also S stars.

The classes are subdivided from 0 to 9. So, for example, Polaris is an F8 star, Sirius is an A1 and the Sun a G2. The scale O through to N gives a non-uniform scale for temperatures.

The Hertzsprung-Russell diagram

The two characteristics of a star which can be seen with the naked eye, its brightness and its colour, thus form the basis of two measures: absolute magnitude and temperature (or spectral class).

These give the axes for a particularly useful type of scatter diagram, the Hertzsprung-Russell (HR) diagram. The vertical scale is absolute magnitude, the low numbers (corresponding to brighter stars) are placed above the high numbers. The horizontal scale represents temperature. In one form of HR diagram, it is based on the sequence of spectral classes; however, it is common to omit the extremes of the range, for example W on the left and R, N and S on the right. Bright, hot stars are found at the top left of the diagram.

Fig. 4 shows typical stars plotted on an HR diagram. It will be seen that most of them lie in a band, which is approximately a straight line running from the top left to the bottom right. However, a few lie elsewhere.

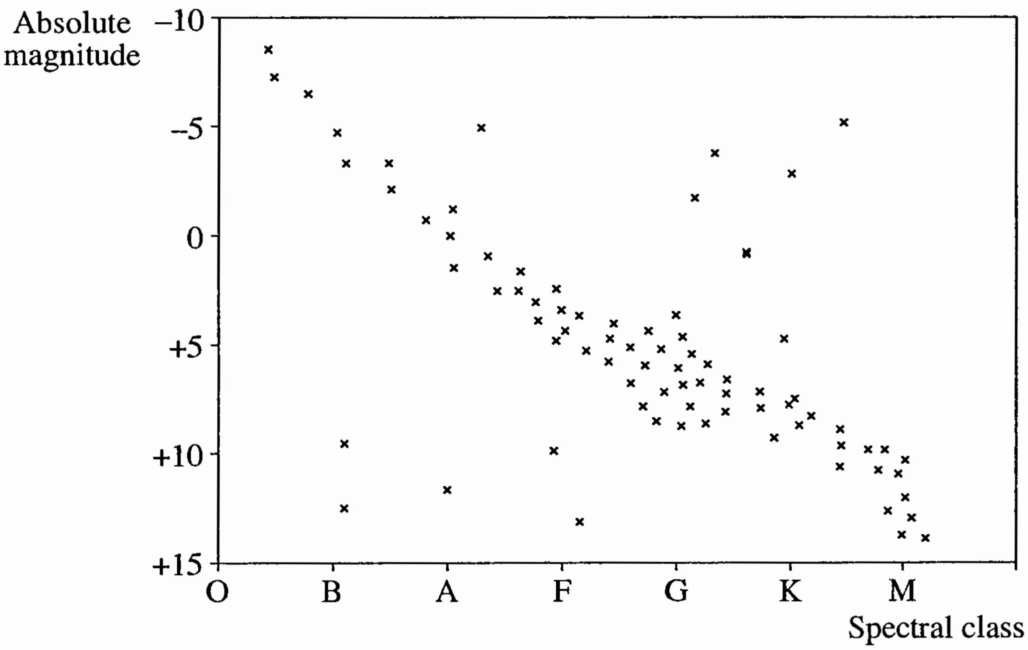


Fig. 4

The band is called the *main sequence* and the stars in it are called main sequence stars. They are in the phase of their lives when they are converting hydrogen to helium by nuclear fusion.

Stars are formed when clouds of gas, mostly hydrogen, condense. In this process, gravitational potential energy is converted into heat and so the future star gets hotter. Eventually it reaches the point where it is sufficiently hot and dense for nuclear fusion to begin. At that point it becomes a star at the bottom right of the main sequence.

As the star gets older, it becomes hotter and brighter, and so its position on the main sequence moves up and to the left. This is a slow process; most stars remain on the main sequence for thousands of millions of years.

Eventually, however, a star reaches the stage where all the hydrogen has been converted into helium. Other fusion reactions then take place, producing heavier elements. At this stage, the star expands and it becomes cooler. Its position on the HR diagram moves well to the right of the main sequence. Such a star is called a *Red Giant*. It is estimated that this will happen to the Sun in about 5000 million years' time. At that point it may well engulf the Earth.

Fig. 5 illustrates the life of a typical star. Larger stars travel further up the main sequence before leaving it. While in its Red Giant phase, a star expels most of its material in a series of explosions. Eventually all that remains is a small, dense core. In this final state, it becomes a *White Dwarf* and occupies a position at the bottom left of the HR diagram.

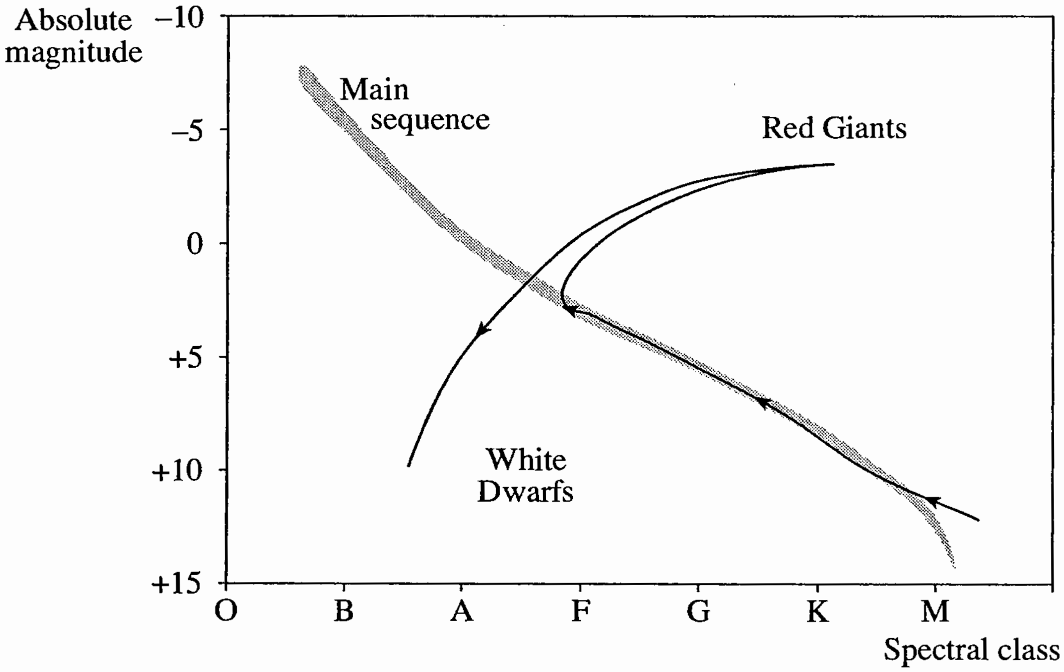


Fig. 5

Mark Scheme

2603A
MEI P3 Jan 2005 Mark Scheme post-coordination
General Instructions

1. (a) Please mark in red and award part marks on the right side of the script, level with the work that has earned them.
   (b) If a part of a question is completely correct, or only *one* accuracy mark has been lost, the total mark or slightly reduced mark should be put in the margin at the end of the section, shown as, for example, 7 or 7 – 1, without any ringing. Otherwise, part marks should be shown as in the mark scheme, as M1, A1, B1, etc.
   (c) The total mark for the question should be put in the right hand margin at the end of each question, and ringed.

2. Every page of the script should show evidence that it has been assessed, even if the work has scored no marks.

3. Do not assume that, because an answer is correct, so is the intermediate working; nor that, because an answer is wrong, no marks have been earned.

4. Errors, slips, etc. should be marked clearly where they first occur by underlining or ringing. Missing work should be indicated by a caret (∧).
   - For correct work, use ✓,
   - For incorrect work, use X,
   - For correct work after and error, use ✓ ‵
   - For error in follow through work, use ✓ ‵‵

5. An 'M' mark is earned for a correct method (or equivalent method) for that part of the question. A method may contain incorrect working, but there must be sufficient evidence that, if correct, it would have given the correct answer.

   An 'A' mark is earned for accuracy, but cannot be awarded if the corresponding M mark has not be earned. An A mark shown as A1 f.t. or A1 ✓ shows that the mark has been awarded following through on a previous error.

   A 'B' mark is an accuracy mark awarded independently of any M mark.

   'E' marks are accuracy marks dependent on an M mark, used as a reminder that the answer has been given in the question and must be fully justified.

6. If a question is misread or misunderstood in such a way that the nature and difficulty of the question is unaltered, follow the work through, awarding all marks earned, but deducting one mark once, shown as MR – 1, from any accuracy or independent marks earned in the affected work. If the question is made easier by the misread, then deduct more marks appropriately.

7. Mark deleted work if it has not been replaced. If it has been replaced, ignore the deleted work and mark the replacement.

8. Other abbreviations:
   c.a.o.: correct answer only
   b.o.d. : benefit of doubt (where full work is not shown)
   X
   | : work of no mark value between crosses
   X
   s.o.i. : seen or implied
   s.c. : special case (as defined in the mark scheme)
   w.w.w : without wrong working

Procedure

1. Before the Examiners' Meeting, mark at least 10 scripts of different standards and bring them with you to the meeting. List any problems which have occurred or that you can foresee.

2. As soon as possible after the meeting, mark the 3 benchmark scripts given out at the Examiners' Meeting, together with another 7 scripts, and send these to your team leader. Keep a record of the marks, and enclose with your scripts a stamped addressed envelope for their return. Your team leader will contact you by telephone or email as soon as possible with any comments. You must ensure that the corrected marks are entered on to the mark sheet.

3. About a week later, you should send a sample of about 40 scripts, covering a range of standards. Again, keep a record of the marks, and enclose with your scripts a stamped addressed envelope for their return. Your team leader will return these scripts to you, and you should amend the marks on the mark sheets as necessary. Your team leader will contact you and, provided all is well, authorise you to send off mark sheets and scripts for batch 1 before the batch 1 deadline.

4. Towards the end of the marking period, contact your team leader, who will ask for a batch 2 of about 80 scripts, from complete centres. These scripts will not be returned to you – you should record your marks on the mark sheets and send these off to the Board separately.

5. Please contact your team leader by telephone or email in case of difficulty. Contact addresses and telephone numbers will be found in your examiner packs.

1 (a) $\frac{5}{(x-2)(x^2+1)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1}$ $\Rightarrow\ 5 = A(x^2+1) + (Bx+C)(x-2)$ $x = 2 \Rightarrow\ 5 = 5A \ \Rightarrow\ A = 1$ coefft of x^2: $0 = A + B \ \Rightarrow B = -1$ coefft of x^0: $5 = A - 2C = 1 - 2C$ $\Rightarrow\ 2C = -4,\ C = -2$ $\Rightarrow\ \frac{5}{(x-2)(x^2+1)} = \frac{1}{x-2} - \frac{x+2}{x^2+1}$	M1 M1 A1 A1 A1 [5]	(Equating numerators,s.o.i. Condone (absence of brackets if subsequent (working is correct substituting $x = 2$ or equating coeffs;independent of first M $A = 1$ $B = -1$ $C = -2$ SC Cover-up rule followed by A=1 and nothing more is M1 A1
(b) $\sin 30° = \frac{1}{2}$ $\cos 30° = \frac{\sqrt{3}}{2}$ $\Rightarrow\ \sin(x+30) = \sin x \cos 30 + \cos x \sin 30$	B1 B1	$\frac{1}{2}, \frac{\sqrt{3}}{2}$ compound angle formula
$= \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x$ $\sqrt{3}\sin x + \cos x = 1$ $\Rightarrow\ 2\sin(x + 30°) = 1$	 B1	 s.o.i.
or, by re-starting $\cos x + \sqrt{3}\sin x = R\sin(x+\alpha)$ $= R(\sin x \cos\alpha + \cos x \sin\alpha)$ $\Rightarrow\ R\cos\alpha = \sqrt{3}$, $R\sin\alpha = 1$ $\Rightarrow\ R^2 = \sqrt{3}^2 + 1^2 = 4,\ R = 2$ $\tan\alpha = 1/\sqrt{3},\ \alpha = 30$ $\Rightarrow\ 2\sin(x+30) = 1$	 B1	
$\Rightarrow\ \sin(x+30) = ½$ $\Rightarrow\ x + 30° = 30°, 150°$ $\Rightarrow\ x = 0, 120°$	M1 A1 A1 [6]	Solving for their $(x+\alpha)$ Or A1 for 30° and 0°
(c) $x = 1 + t^2 \quad \Rightarrow \mathrm{d}x/\mathrm{d}t = 2t$ $y = t^3 \quad \Rightarrow \mathrm{d}y/\mathrm{d}t = 3t^2$ $\Rightarrow\ \mathrm{d}y/\mathrm{d}x = \frac{dy/dt}{dx/dt}$ $= 3t/2$ When $x = 2$ and $y = 1$, $t = 1$ $\Rightarrow\ \mathrm{d}y/\mathrm{d}x = 3/2$	M1 A1 M1 A1cao [4] [15]	$\mathrm{d}y/\mathrm{d}x = \frac{dy/dt}{dx/dt}$ $3t/2$ accept $3t^2/2t$ $t = 1$ or both $1+t^2=2$ and $t^3=1$ and attempt to solve s.o.i. by $\mathrm{d}y/\mathrm{d}x=3/2$

2 (i) $x\sec^2 x = \frac{x}{\cos^2 x} = \frac{x}{(1-\frac{1}{2}x^2)^2}$ $= x(1 - ½\, x^2)^{-2}$ * $= x(1 + (-2)(-½\, x^2) + \ldots)$ $= x + x^3 + \ldots$*	M1 E1 M1 E1 [4]	$\cos x = 1 - ½\, x^2$ soi and used Binomial expansion - ignore subsequent terms Independent of first E1
(ii) $\int_0^{0.1}(x+x^3)dx = \left[\frac{1}{2}x^2 + \frac{1}{4}x^4\right]_0^{0.1}$ $= 0.005025$	M1 A1cao [2]	$\left[\frac{1}{2}x^2 + \frac{1}{4}x^4\right]$ Accept 0.00503 or 5.03×10^{-3} or better Accept one fraction of equivalent accuracy
(iii) $\frac{d}{dx}(\frac{\sin x}{\cos x}) = \frac{\cos x.\cos x - \sin x.(-\sin x)}{\cos^2 x}$ $= \frac{1}{\cos^2 x}$ $= \sec^2 x$	M1 A1 E1 [3]	quotient rule numerator = 1 www
(iv) $\int_0^{0.1} x\sec^2 x dx = \int_0^{0.1} x\frac{d}{dx}(\tan x)dx$ $= \left[x\tan x\right]_0^{0.1} - \int_0^{0.1}\tan x dx$ $= 0.1\tan 0.1 + \left[\ln\cos x\right]_0^{0.1}$ $= 0.1\tan 0.1 + \ln\cos 0.1$ $= 0.005025\ldots$ Approximation is accurate to 4 s.f.	M1 A1 A1 M1 A1cao B1 [6] [15]	$u = x$, $v = \tan x$ $\left[x\tan x\right] - \int \tan x dx$ $\ln\cos x$ substituting limits; allow even if calculator is used in °mode. Answers to (ii) and (iv) must be correct to earn this B1

3 (i) $\frac{dy}{dx} = -\frac{4x}{y}$ $\Rightarrow \int y\,dy = \int -4x\,dx$ $\Rightarrow \frac{1}{2}y^2 = -2x^2 + c$ substituting $x = 0, y = 2$ $\Rightarrow 2 = c$ $\Rightarrow \frac{1}{2}y^2 = -2x^2 + 2$ $\Rightarrow 4x^2 + y^2 = 4$ *	M1 A1 M1 E1	Separating the variables Any correct form. calculating c. Condone the absence of c for the first A1 but it must be present in the solution to earn this M1
or $4x^2 + y^2 = 4$ when $x = 0, y = 2, 4x^2 + y^2 = 0 + 4 = 4$ Differentiating implicitly: $8x + 2y\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{8x}{2y} = -\frac{4x}{y}$	E1 M1 A1 E1 [4]	verifying (0, 2) is on curve implicit differentiation
(ii) $x = \cos\theta$, $y = 2\sin\theta$ $\Rightarrow 4x^2 + y^2 = 4\cos^2\theta + 4\sin^2\theta$ $= 4(\cos^2\theta + \sin^2\theta) = 4$ Ellipse	M1 E1 B1 [3]	 Some intermediate step must be seen.
(iii) $\tan\alpha = y/x$ from the differential equation $\tan\beta = -4x/y$ $\Rightarrow \tan\alpha\tan\beta = \frac{y}{x}\times(-\frac{4x}{y}) = -4$ *	B1 E1 [2]	For $\tan\alpha = y/x$ <u>or</u> $\tan\beta = -4x/y$ For <u>both</u> above and the result
(iv) $\tan\alpha\tan\beta = \tan\alpha\tan 2\alpha$ $= \tan\alpha.\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\tan^2\alpha}{1-\tan^2\alpha}$ $\Rightarrow \frac{2\tan^2\alpha}{1-\tan^2\alpha} = -4$ $\Rightarrow 2\tan^2\alpha = -4 + 4\tan^2\alpha$ $\Rightarrow 4 = 2\tan^2\alpha$ $\Rightarrow \tan^2\alpha = 2$ * $\Rightarrow \tan\alpha = \sqrt{2}$ $\Rightarrow \alpha = 54.73\ldots°$ $= 55°$	 B1 M1 M1 E1 M1 A1cao [6]	 Forming equation $\tan\alpha$ (their $\tan 2\alpha$)$=-4$ Simplifying their equation to the form $\tan^2\alpha$ = const.

4 (i) $\overrightarrow{BC} = \begin{pmatrix}1.4\\-0.2\\-1\end{pmatrix}$, $\overrightarrow{BF} = \begin{pmatrix}0.2\\1.4\\-1\end{pmatrix}$	B1 [1]	Accept row vectors
(ii) BC = $\sqrt{(1.4^2 + 0.2^2 + 1^2)}$ = BF so triangle BCF is isosceles	M1 A1w.w.w. [2]	Length formula applied to BC and BF SCB1 for <u>convincing</u> verbal argument
(iii) $\mathbf{n_1}.\overrightarrow{BC} = \begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}1.4\\-0.2\\-1\end{pmatrix} = 5.6 - 0.6 - 5 = 0$ $\mathbf{n_1}.\overrightarrow{BF} = \begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}0.2\\1.4\\-1\end{pmatrix} = 0.8 + 4.2 - 5 = 0$ Equation of plane is therefore $4x + 3y + 5z = d$ Substituting $x = 4$, $y = 3$, $z = 2$: $\Rightarrow$ $d = 16 + 9 + 10 = 35$ $\Rightarrow$ plane is $4x + 3y + 5z = 35$.	M1 A1 A1 M1 A1 [5]	Scalar product with $\overrightarrow{BC}$ or with CF = $\begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}-1.2\\1.6\\0\end{pmatrix}$ = –4.8+4.8 or with $\overrightarrow{BF}$ Allow one error if all three are done. OR M1 correct form of vector equation A1 correct vector equation M1 eliminate parameters A1 correct cartesian equation A1 confirm the normal
(iv) At A (0, 0, 2): 3×0 – 4×0 + 5×2 = 10 At B (4, 3, 2): 3×4 – 4×3 + 5×2 = 10 At D (6, 2, 0): 3×6 – 4×2 + 5×0 = 10 $\mathbf{n}_2 = \begin{pmatrix}3\\-4\\5\end{pmatrix}$	M1 A1 B1 [3]	substituting one set of coordinates into equation all three done correctly OR M1 correct form of vector equation and elimination of parameters A1 correct Cartesian equation
(v) $\cos\theta = \dfrac{\mathbf{n_1}.\mathbf{n}_2}{\lvert\mathbf{n_1}\rvert.\lvert\mathbf{n}_2\rvert}$ $= \dfrac{\begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}3\\-4\\5\end{pmatrix}}{\sqrt{50}.\sqrt{50}} = \dfrac{25}{50} = \dfrac{1}{2}$ $\Rightarrow$ $\theta = 60°$.	M1 A1ft A1cao A1 [4]	Use of $\mathbf{n_1}$ and their $\mathbf{n_2}$ scalar product = 25 ft their $\mathbf{n_2}$ cos θ = ½ allow 120°

Mark scheme

1. Points to be plotted **B1, B1, B1**

Star	Type	Abs. Mag.
The Sun	G2	4.8
Sirius	A1	1.4
Polaris	F8	-4.6

2. $m_2 - m_1 = -2.5 \times \log_{10}\left(\frac{E_2}{E_1}\right)$

Take star 1 as Bellatrix and star 2 as Arcturus, **M1**

so $m_1 = 1.64$, $\frac{E_2}{E_1} = 4.7$ and m_2 is to be found

$m_2 - 1.64 = -2.5 \times \log_{10}(4.70)$

The apparent magnitude of Arcturus is -0.04 **A1**

3. (i) (*A*) Using $d = \frac{b}{2\tan\alpha}$) **M1**

with $b = 2 \times (1.5 \times 10^8)$)

and $\alpha = 1''$) **A1**

$\Rightarrow d = 3.093... \times 10^{13}$ in kilometres **A1**

(i) (*B*) Convert to light years by dividing by $3 \times 10^5 \times 60 \times 60 \times 24 \times 365$ **M1**

$\Rightarrow d = 3.27$ **A1**

(ii) The values used for the radius of the earth's orbit, the speed of light and the length of a year, were approximate. Also the Earth's orbit is assumed to be circular. Any <u>one</u> reason is sufficient for **B1**

4. $M_{Abs} = M_{App} - 2.5\log_{10}\left(\frac{d^2}{100}\right)$

$M_{Abs} = M_{App} - 2.5\log_{10}\left(\frac{d}{10}\right)^2$

$M_{Abs} = M_{App} - 2.5 \times 2 \times \log_{10}\left(\frac{d}{10}\right)$ **M1**

$M_{Abs} = M_{App} - 5 \times [\log_{10} d - \log_{10} 10]$ **M1**

$M_{Abs} = M_{App} - 5 \times [\log_{10} d - 1]$

$M_{Abs} = M_{App} + 5 - 5\log_{10} d$. **E1**

5. Sirius has a large apparent magnitude because it is quite close to us, even though it does not have a very large absolute magnitude. **B1**

or an equivalent comment, e.g.

O and B stars on the main sequence have higher absolute magnitudes.

Examiner’s Report